F.Y.B.Sc.(Part-I)(With Credits)-Regular-Semester 2012 Sem. I

MAT-102 - Mathematics-II (Differential and Integral Calculus)

P. Pages : 3 **GUG/W/16/3318**

Time : Three Hours * 4 6 1 8 * Max. Marks : 60

Notes : 1. Solve **all five** questions.
2. Q. No. 1 to 4 have an alternative. Solve each question in full or its alternative in full.
3. All question carry equal marks.

UNIT – I

1. a) Let f(x) and g(x) be defined at all points of an interval [a, b] except possibly at $x_0 \in [a, b]$ if $\lim_{x \to x_0} f(x) = A$ and $\lim_{x \to x_0} g(x) = B$. Then prove that $\lim_{x \to x_0} \{f(x) + g(x)\} = A + B$. **6**

b) Show that the function **6**

$$f(x) = (1+2x)^{1/x}, \quad x \neq 0$$
$$= e^2 \qquad , \quad x = 0$$

is continuous at x = 0.

OR

c) If a real function f defined on [a, b] is continuous on [a, b] and differentiable on (a, b) then prove that there exists at least one point $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. **6**

d) Show that $\frac{x}{1+x^2} < \tan^{-1} x < x \quad \forall\ x > 0$. **6**

UNIT – II

2. a) If $y = \left(x + \sqrt{1+x^2}\right)^m$ then show that **6**

$$\left(1+x^2\right) y_{n+2} + (2n+1) x y_{n+1} + \left(n^2 - m^2\right) y_n = 0$$

b) If $\cos^{-1}(y/b) = \ell n (x/n)^n$, prove that **6**

$$x^2 y_{n+2} + (2n+1) x y_{n+1} + 2n^2 y_n = 0$$

OR

c) f(x) and g(x) are differentiable in (a, b) except possibly at a point $x_0 \in (a\, b)$ and $f(x_0) = g(x_0) = 0$ then prove that $\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$, if limit exist. **6**

d) Prove that $\lim_{x\to 1}\left[\frac{1}{\ell ux}-\frac{x}{x-1}\right]=-1/2$. **6**

UNIT – III

3. a) Integrate $\int \frac{dx}{(x-1)\sqrt{x^2+x+1}}$, $x>1$ **6**

b) Evaluate $\int (2x+5)\sqrt{x^2+3x+1}dx$ **6**

OR

c) If $In=\int \text{cosec}^n x\,dx$ then prove that $In=-\frac{1}{n-1}\text{cosec}^{n-2}x\cot x+\frac{n-2}{n-1}I_{n-2}$ **4+2**

hence evaluate $\int \frac{dx}{\sin^3 x}$.

d) If $\phi(n)=\int_0^{\pi/4}\tan^n x\,dx$ then show that $\phi(n)+\phi(n-2)=\frac{1}{n-1}$ and find the value of $\phi(5)$. **6**

UNIT – IV

4. a) Prove that $\lceil(n+1)=n\ \lceil n$ **6**

b) Define Beta function and prove that **2+4**

$$\beta(m,n)=2\int_0^{\pi/2}\sin^{2m-1}\theta.\cos^{2n-1}\theta\ d\theta$$

OR

c) Prove that **6**

$$B(m,n)=\frac{\lceil m\ \lceil n}{\lceil m+n}$$

d) Prove that $\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta=\pi/\sqrt{2}$. **6**

5. Attempt **any six.**

a) Show that $\lim_{x\to 0} f(x)$ does not exists if **2**

$$f(x)=\frac{|x|}{x},\ x\neq 0$$
$$=0,\ x=0$$

b) State Rolle's theorem. **2**

c) If $y = A\sin mx + B\cos mx$. **2**
Prove that $y_2 + m^2 y = 0$

d) If $y = e^{ax} \sin bx$, **2**
prove that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$

e) Integrate $\int \frac{dx}{\sqrt{3 + 4x - 4x^2}}$ **2**

f) Evaluate $\int \sqrt{2x - x^2}\ dx$ **2**

g) Evaluate $\int_0^{\infty} \frac{x^3}{(1+x)^7}\ dx$ **2**

h) Evaluate : **2**
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