B.Sc. (with Credits)-Regular-Semester 2012 Sem VI

B.Sc.4532 - Mathematics – II Paper - VI (Optional)
(Number Theory and Discrete Mathematics)

P. Pages : 2 **GUG/W/16/5649**

Time : Three Hours * 0 1 1 3 * Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. Question 1 and 4 has an alternative solve each question in full or its alternative in full.
3. Each questions carries equal marks.

UNIT - I

1. a) Prove that all solutions of $3x+5y=1$ can be written in the form $x=2+5y,\ y=-1-3t$ also in the form of $x=2-5t,\ y=-1+3t$. **6**

b) Prove that $ax+by=a+c$ is solvable iff $ax+by=c$ is solvable. **6**

OR

c) If p is a prime then $(p-1)!\equiv -1(\mathrm{mod}\,p)$. **6**

d) Find all solutions in positive integers $5x+3y=52$. **6**

UNIT - II

2. a) Solve $49x\equiv 47(\mathrm{mod}\,81)$ **6**

b) Solve the following simultaneous congruence **6**

$x\equiv 4(\mathrm{mod}\,12)$

$x\equiv 7(\mathrm{mod}\,21)$

$x\equiv 10(\mathrm{mod}\,15)$

OR

c) Solve the congruence $2x^2-3x+1\equiv 0\ (\mathrm{mod}\,105)$. **6**

d) Find the solution of the system of congruence **6**

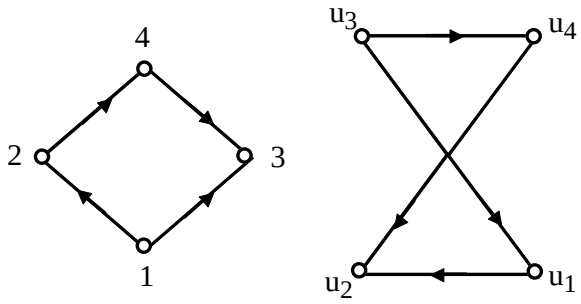
$x\equiv 7(\mathrm{mod}\,16)$

$x\equiv 35(\mathrm{mod}\,36)$

$x\equiv 5(\mathrm{mod}\,42)$

UNIT - III

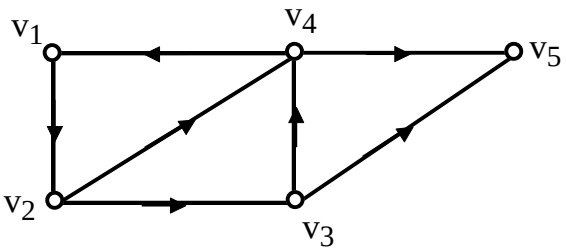
3. a) Show that following graph are isomorphic. **6**



b) In a simple diagraph, the length of any elementary path is less than or equal to (n - 1) where n is the number of nodes in the graph. Similarly the length of any elementary cycle does not exceed n. **6**

OR

c) Find all the indegrees and outdegrees of the diagraph given below. Give all the elementary cycles of this graph. List all the nodes which are reachable from another node of the graph. **6**



d) Define : Trees, Directed trees, ordered trees, path length of a vertex, forest, Binary trees. **6**

UNIT - IV

4. a) Let $(L, \leq)$ be a lattice for any $a, b, c \in L$ following hold : **6**

$$b \leq c \Rightarrow \begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$$

b) In a distributive lattice the complement of a element is unique. **6**

OR

c) Prove that the Boolean identities : **6**

i) $a \oplus (a' * b) = a \oplus b$ ii) $a * (a' \oplus b) = a * b$

iii) $(a * b) \oplus (a * b') = a$

d) Simplify the following expression **6**

i) $(a * c) \oplus c[(b \oplus b') * c]$ ii) $(a' * b' * c') \oplus (a * b' * c) \oplus (a * b' * c')$

5. Attempt **any six.**

a) State Wilson Theorem. **2**

b) State Fermat's theorem. **2**

c) Solve the congruence $6x \equiv 3 (\text{mod} 9)$ **2**

d) Define a linear congruence. **2**

e) Define a node - base. **2**

f) Define indegree and outdegree of node. **2**

g) Define external and internal node. **2**

h) Define strongly connected and weakly connected. **2**

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