B.Sc. (With Credits)-Regular-Semester 2012 Sem V

B.Sc.3528 - Mathematics : Paper-I (Linear Algebra)

P. Pages : 2 **GUG/W/16/3371**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) If the function f(z) = u+iv be analytic in the domain D. Prove that the families of curves $u(x, y) = C_1$ and $v(x, y) = C_2$ form an orthogonal system, where C_1 and C_2 are arbitrary constants. **6**

b) Prove that $u = y^3 - 3x^2y$ is a harmonic function. Find its conjugate and the corresponding analytic function f(z) in terms of z. **6**

OR

c) Find the bilinear transformation which maps the points z=1, i, -1 into the points w=i, 0, -i. **6**

d) Prove that every bilinear transformation with a single non-infinite fixed point α can be put in the form $\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + k$, where k is a constant. **6**

UNIT – II

2. a) Prove that intersection of two subspaces of a vector space is a subspace. **6**

b) Let S be a nonempty subset of a vector space V. Prove that L(S) is the smallest subspace of V containing S. Where L(S) is the Linear span of S. **6**

OR

c) Prove that the set {(1, 0, 0), (1, 1, 0), (1, 1, 1)} is a basis of V_3. **6**

d) If U and W are finite dimensional subspace of a vector space V. **6**
Prove that : $\dim(U+W) = \dim U + \dim W - \dim(U \cap W)$.

UNIT – III

3. a) If $T: V_2 \to V_4$ be a linear map defined by T(1, 1) = (0, 1, 0, 0) and T(1, -1) = (1, 0, 0, 0). Where {(1, 1), (1, -1)} is a basis of V_2. Find T(x, y). **6**

b) Let U, V be vector spaces over the same field F. Prove that a function $T: U \to V$ is linear iff $T(\alpha u+v) = \alpha T(u)+T(v)$, $\forall u, v \in U$ and $\alpha \in F$. **6**

OR

c) Let $T: V_4 \to V_3$ be a linear map defined by $Te_1 = (1,1,1)$, $Te_2 = (1,-1,1)$, $Te_3 = (1,0,0)$ and $Te_4 = (1,0,1)$. Verify Rank-nullity theorem. **6**

d) Let B = {(1, 1, 1) (1, 0, 1) (0, 0, 1) be a basis for V_3. Find the coordinate vector of $(2,3,4) \in V_3$ relative to basis B. **6**

UNIT – IV

4. a) Let V be an inner product space over F. If $u, v \in V$. Prove that : $|(u, v)| \le \|u\| \cdot \|v\|$. **6**

b) Let $\{x_1, x_2, ----, x_n\}$ be an orthogonal set. **6**
Prove that : $\|x_1 + x_2 + ----- + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + ------ + \|x_n\|^2$

OR

c) If $\{w_1, ---, w_m\}$ is an orthonormal set in V. Prove that : $\sum_{i=1}^{m} |(w_i, v)|^2 \le \|v\|^2$ for any $v \in V$. **6**

d) Find the orthonormal basis of $P_2[-1, 1]$ starting from the basis $\{1, x, x^2\}$ using the inner product defined by $(f, g) = \int_{-1}^{1} f(x) g(x) dx$. **6**

5. Attempt **any six.**

a) Define a Harmonic and conjugate functions. **2**

b) Let $f(z) = \frac{1+z}{1-z}$. Determine where f(z) is non analytic. **2**

c) Let V be a vector space over F. **2**
Prove that : $(-\alpha)v = -(\alpha v) \forall \alpha \in F, \forall v \in V$.

d) Define a linear span of a subset of a vector space. **2**

e) Show that $T: R^2 \to R^2$, defined by $T(x, y) = (x+1, y+2)$, is not a linear map. **2**

f) Define a isomorphism of vector spaces. **2**

g) Define a orthogonal complement in a vector space. **2**

h) Let V be an inner product space over F. **2**
Prove that : $\|\alpha u\| = |\alpha| \|u\|, \forall \alpha \in F, u \in V$.

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