B.Sc. (With Credits)-Regular-Semester 2012 Sem III

B.Sc.23111 - Mathematics Paper - I
(Advanced Calculus And Group Theory)

P. Pages : 2 **GUG/W/16/3344**

Time : Three Hours Max. Marks : 60

Notes :
1. Solve **all five** questions.
2. Q. No. 1 to 4 have an alternative. Solve each question in full or its alternative in full.
3. All questions carry equal marks.

UNIT – I

1. a) If G is a finite group and H is a subgroup of G then prove that O (H) is a divisor of O (G). **2+4**

b) Prove that a group G is abelian iff $a, b \in G$. **6**

$(a \cdot b)^2 = a^2 \cdot b^2$

OR

c) Let H is subgroup of a group G then prove that any two left cosets of H in G are either identical or disjoint. **6**

d) Prove that a subgroup H of a group G is normal subgroup of G iff each left coset of H in G is a right coset of H in G. **6**

UNIT – II

2. a) Prove that, every permutation is a product of 2 – cycles or transposition. **6**

b) Prove that $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8)^4 = (15)(26)(37)(48)$. **6**

OR

c) Let $f : G \to G'$ is homomorphism. Prove that Kernel of it is normal subgroup of G. **6**

d) Let $f : G \to G'$ is homomorphism and e, e' are identities in G and G' respectively then prove that $f(e) = e'$ and $f(a^{-1}) = (f(a))^{-1}$; $a \in G$. **3+3**

UNIT – III

3. a) Let $Z = f(x, y) = \frac{y(x^2 + y^2)}{y^2 + (x^2 + y^2)^2}$. Show that f has limit 0 as $(x, y) \to (0, 0)$ on a ray $x = at$, $y = bt$ but f does not have limit 0 as $(x, y) \to (0, 0)$. **6**

b) By using $\in - \delta$ definition of limit prove that **6**

$$\lim_{(x,y)\to(1,1)} \left(x^2 + 2y\right) = 3 \cdot$$

OR

c) The function $f(x, y) = ae^{-bx} \sin(cy - bx)$ where a, b, c are constants satisfies the equation $\frac{\partial t}{\partial y} = K \frac{\partial^2 f}{\partial x^2}$. Find the value of K. **6**

d) If $u = f(x - y, y - z, z - x)$ prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **6**

UNIT – IV

4. a) Verify Euler's theorem on homogeneous function for $u = \log\left(\frac{x+y}{x-y}\right)$. **6**

b) If $x = \Omega \sin\theta \cos\phi$, $y = \Omega \sin\theta \sin\phi$, $z = r\cos\theta$ find $\frac{\partial(x, y, z)}{\partial(\Omega, \theta, \phi)}$. **6**

OR

c) Find absolute maxima, minima values of $f(x, y) = x^3 + y^3 - 3axy$. **6**

d) Use the method of Lagrange Multipliers locate all local maxima and local minima, also find absolute or minimum for $f(x, y, z) = x + y + z$ where $x^2 + y^2 + z^2 = 1$. **6**

5. Attempt **any six.**

a) Prove that, identity of group G is unique. **2**

b) Show that every subgroup of an abelian group is normal. **2**

c) Prove that every cyclic group is abelian. **2**

d) Find the orbit of 2 ; 5 when the permutation is $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 6 & 4 \end{pmatrix}$. **2**

e) Show that $f(x, y) = x - y$ is continuous for all $(x, y) \in R^2$. **2**

f) If $u = e^x(x\cos y - y\sin y)$ find $\frac{\partial u}{\partial x}$. **2**

g) State the second derivative test for maxima minima for a function of two variables. **2**

h) Find stationary points of $f(x, y) = x^2 + y^2 + xy + x - 4y + 5$. **2**

****