B.Sc. (With Credits)-Regular-Semester 2012 Sem II

2SMAT 103 - Mathematics -I

(Vector Calculus, Geometry & Difference Equations)

Paper - III

P. Pages : 2 **GUG/W/16/5583**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Show that $\bar{a}\times(\bar{b}\times\bar{c})$, $\bar{b}\times(\bar{c}\times\bar{a})$, $\bar{c}\times(\bar{a}\times\bar{b})$ are coplanar. **6**

b) The sets $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{a}'$, $\bar{b}'$, $\bar{c}'$ are reciprocal sets of vectors then prove that **6**

$$\bar{a}'=\frac{\bar{b}\times\bar{c}}{[\bar{a}\,\bar{b}\,\bar{c}]},\ \bar{b}'=\frac{\bar{c}\times\bar{a}}{[\bar{a}\,\bar{b}\,\bar{c}]},\ \bar{c}'=\frac{\bar{a}\times\bar{b}}{[\bar{a}\,\bar{b}\,\bar{c}]}$$

OR

c) If $\bar{r}=a\cos t\,\bar{i}+a\sin t\,\bar{j}+a\,t\tan\alpha\,\bar{k}$ then find $|\dot{\bar{r}}\times\ddot{\bar{r}}|$ and $[\dot{\bar{r}}\ \ddot{\bar{r}}\ \dddot{\bar{r}}]$ **6**

d) If $\bar{v}=\bar{w}\times\bar{r}$ then prove that $\bar{w}=\frac{1}{2}\text{curl}\bar{v}$ where $\bar{w}$ is a constant vector. **6**

UNIT - II

2. a) If f(x, y) and g(x, y) are continuous functions defined on a region D of area A. Then prove that **6**

i) $\iint_D cf(x,y)dA=c\iint_D f(x,y)dA,$ c is constant

ii) $\iint_D [f(x,y)+g(x,y)]\,dA=\iint_D f(x,y)dA+\iint_D g(x,y)\,dA.$

b) **6**

Evaluate the integral $\int_0^1\int_0^{\sqrt{1+x^2}}\frac{1}{1+x^2+y^2}\,dx\cdot dy$

OR

c) Evaluate $\iint r.\sin\theta dr.d\theta$ over the area of cardioid $r=a(1-\cos\theta)$ above the initial line. **6**

d) **6**

Evaluate $\int_0^1\int_{x^2}^{2-x} xydy\cdot dx$ by changing the order of integration.

UNIT - III

3. a) Prove that the curves of intersection of two spheres is circle. **6**

b) Find the equation of sphere which touches the sphere $x^2+y^2+z^2+2x-6y+1=0$ at (1, 2, -2) and passes through the point (1, -1, 0) **6**

OR

c) Find the equation of right circular cone generated by revolving a straight line $5y+2z=10,\ x=0$ about z-axis. **6**

d) Find the equation of cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and guiding curves is the ellipse $x^2+2y^2=1,\ z=3$. **6**

UNIT - IV

4. a) From $y_n=a(2)^n+b(-2)^n$, derive a difference equation by eliminating the constants. **6**

b) Solve the equation $y_{n+2}-2\cos\alpha y_{n+1}+y_n=0$ **6**

OR

c) Solve the difference equation $y_{n+3}-5y_{n+2}+3y_{n+1}+9y_n=2^n+3^n$ **6**

d) Solve the equation $y_{k+1}+\frac{1}{4}y_k=\left(\frac{1}{4}\right)^k\ k\geq 0, y(0)=1$ **6**

5. Attempt **any six.**

a) If $\bar{a}=\bar{i}-\bar{j}-\bar{k},\ \bar{b}=\bar{j}\times\bar{k}$ then find $(\bar{a}\times\bar{b})$. **2**

b) If $\phi=|\bar{r}|$ then find grad ϕ. **2**

c) Evaluate $\int_0^1\int_0^x e^u\,dy\,dx$ **2**

d) Prove that $f(x, y)\geq 0$ on $D\Rightarrow\iint_D f(x,y)\,dA\geq 0$ **2**

e) Find the equation of sphere possing through origin and intercepts of a, b, c on the coordinate axes resp. **2**

f) Find the equation of the cone with vertex at the origin and direction cosines of generator satisfying the relation $3\ell^2=4m^2+5n^2=0$. **2**

g) Define the order of difference equation with example. **2**

h) Solve the difference equation $4y_{n+2}-4y_{n+1}+y_n=0$. **2**

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