B.Sc. III (with Credits)-Regular-Semester 2012 Sem VI

B.Sc.4530 - Mathematics Paper-IV
Analysis

P. Pages : 2 **GUG/W/16/5647**

Time : Three Hours Max. Marks : 60

Notes :
1. Solve all the **five** questions.
2. Question 1 to 4 has an alternative. Solve each question in full or its alternative in full.
3. All questions carry equal marks.

UNIT - I

1. a) Show that $d(x, y) = |x - y|, \forall\ x, y \in R$ defines a metric on R. **6**

b) Prove that every neighbourhood is an open set. **6**

OR

c) Prove that $\{S_n\}$ is a Cauchy sequence of real numbers if and only if $\{S_n\}$ is convergent in R i.e. R is a complete metric space. **6**

d) Prove that compact subsets of a metric space are closed. **6**

UNIT - II

2. a) Let f, g be bounded functions defined on [a, b] and p be any partition of [a, b]. Then prove that $U(P, F+g) \le U(P, F) + U(p, g)$ and $L(P, F+g) \ge L(P, F) + L(P, g)$. **6**

b) If f is bounded and integrable function over [a, b] and M, m are the bounds of f over [a, b] then prove that $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$ **6**

OR

c) If f is monotonic in [a, b] then prove that it is integrable on [a, b]. **6**

d) If $f \in R[a, b]$ then prove that $F:[a, b] \to R$ defined by $F(x) = \int_a^x f(t)\,dt$ is continuous on [a, b]. If f is continuous at $x_0 \in [a, b]$ then also prove that F is differentiable at x_0 with $F'(x_0) = f(x_0)$ and if f is continuous on [a, b]. Then F is differentiable on [a, b] with $F'(x) = f(x)\ \forall\ x \in [a, b]$. **6**

UNIT - III

3. a) Prove that **6**

i) $\int_c \frac{dz}{z-a} = 2\pi i$

ii) $\int_c (z-a)^n dz = 0$, n , any integer $\ne -1$ where c is the circle $|z-a| = r$.

b) If a function f(z) is analytic in a simply connected domain D, then prove that $\int_c f(z)dz = 0$ for every simple closed curve C in D.

OR

c) Evaluate $\int_c \frac{15z+9}{z(z^2-9)}dz$, where c is the circle $|z-1| = 3$.

d) Evaluate $\int_c \frac{z-3}{z^2+2z+5}dz$, where c is the circle $|z+1+i| = 2$ by using Cauchy residue theorem.

UNIT - IV

4. a) Find the finite Fourier sine and cosine transforms of f(x) = sin ax in the interval $(0, \pi)$. **6**

b) Find the finite sine and cosine transform of mx, $0 < x < \ell$. **6**

OR

c) Show that the Fourier transform of $f(x)e^{-ax^2}$ is $\sqrt{\pi/a}\ e^{-\lambda^2/4a}$. **6**

d) Show that Fourier cosine transform of $f(x) = e^{-x^2}$ is $\frac{1}{\sqrt{2}}e^{-\lambda^2/4}$ **6**

5. Attempt **any six.**

a) Define complete metric space. **2**

b) Define open and closed sphere. **2**

c) Prove that $M(b-a) \le L(p, f) \le U(p, f) \le M(b-a)$ where M and m denote the sup and in f of f(x) in I = [a, b]. **2**

d) Prove that inequalities $\int_0^1 e^{x^2} dx > 0$ **2**

e) Evaluate $\int_c \bar{z}\,dz$, where c is the unit circle |z| = 1. **2**

f) Show that $\int_c dz$ and $\int_c z\,dz$ vanish on the smooth closed contour c. **2**

g) Define finite Fourier cosine transform. **2**

h) State convolution theorem for Fourier transform. **2**

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