B.Sc. II (With Credits)-Regular-Semester 2012 Sem IV

B.Sc.24112 - Mathematics -II (Classical Mechanics & Statics)

Paper - IV

P. Pages : 2 **GUG/W/16/5611**

Time : Three Hours Max. Marks : 60

Notes :
1. Solve all the **five** questions.
2. Question 1 to 4 has an alternative solve each question in full or its alternative in full.
3. All questions carry equal marks.

UNIT - I

1. a) Prove that a system of coplanar forces acting upon a rigid body will be in equilibrium if the algebraic sum of the moments of all the forces about each of three non-collinear points is zero. **6**

b) Four equal rods of length 2a and W are smoothly joined to form a rhombus ABCD which is kept in shape by a light rod BD. Angle BAD is 60^o and the rhombus is suspended in a vertical plane from A. Find the thrust in BD. **6**

OR

c) Obtain the Cartesian equation of the uniform catenary $y = c\cosh\frac{x}{c}$. **6**

d) A uniform chain of length 2ℓ has its end fixed at two points in the same level. If the sag at the middle is h, then prove that the span is $\frac{\ell^2 - h^2}{h}\log\frac{\ell + h}{\ell - h}$ **6**

UNIT - II

2. a) Discuss the motion of a particle in a plane by using polar coordinates. **6**

b) A bead is sliding on a uniformly rotating wire in a force-free space. Show that the acceleration of the bead is $\ddot{r} = rw^2$, where w is the angular velocity of rotation. **6**

OR

c) Show that the rate of energy dissipation due to friction is 2R. **6**

d) Obtain the equation of motion for a particle falling vertically under the influence of gravity when frictional forces obtainable from a dissipation function $\frac{1}{2}kv^2$ are present. Integrate the equation to obtain the velocity as a function of time and show that the maximum possible velocity for all from rest is $v = mg/k$. **6**

UNIT - III

3. a) Prove that the problem of motion of two masses interacting only with one another always be reduced to a problem of the motion of a single mass. **6**

b) Prove that for a central force field F, the path of a particle of mass m is given by **6**

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{h^2u^2} F\left(\frac{1}{u}\right), u = \frac{1}{r}$$

OR

c) If the potential energy is a homogeneous function of degree -1 in the radius vector $\bar{r}_i$, then the motion of a conservative system takes place in a finite region of space only if the total energy is negative. **6**

d) For a central force field, show that Kepler's second law is a consequence of the conservation of angular momentum. **6**

UNIT - IV

4. a) Use Hamilton's principle to find the equation of motion of a particle of mass moving in space in a conservative force field $\bar{F}$. **6**

b) Prove that Hamilton's principle is a necessary and sufficient condition for Lagrange's equation. **6**

OR

c) A particle of mass falls a given distance S_0 in time to = $\sqrt{2S_0 \mid g}$ and the distance fallen in time t is given by $S = at + bt^2$, where a and b are constants. Show that Hamilton's principle is valid only when a = 0 and b = g/2. **6**

d) Show that for a single particle system, the least action principle yields. **6**

$\Delta \int \sqrt{2m(H-V)}\, ds = 0$ where $ds = |d\bar{r}|$.

5. Solve **any six.** **12**

a) Obtain $S = c \sinh \frac{x}{c}$

b) Show that $\rho = c \sec^2 \psi$, where ρ is the radius of curvature of the curve at any point P.

c) Write down the Lagrangian and equation of motion for a mass m suspended by a spring of force constant k and allowed to swing vertically.

d) Define Rayleigh's dissipation function.

e) State Kepler's first law.

f) State Virial theorem.

g) Prove that a cyclic coordinate will be absent in Hamiltonian.

h) Show that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$

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