

B.Sc. II (With Credits)-Regular-Semester 2012 Sem IV
B.Sc.24111 - Mathematics -I (Abstract Algebra & Differential Equations)
Paper - III

P. Pages : 2

Time : Three Hours



GUG/W/16/5610

Max. Marks : 60

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- Notes : 1. Solve all **five** questions.
2. Question 1 to 4 has an alternative. Solve each question in full or its alternative in full.
3. All questions carry equal marks.

UNIT - I

1. a) Let G is a group for fixed $g \in G$ define $T_g : G \rightarrow G$ such that $T_g(x) = g^{-1}xg$, $x \in G$. **6**
Prove that T_g is automorphism of G .

b) Let H and K are subgroups of a group G prove that HK is subgroup of G iff $HK=KH$. **6**

OR

c) Let G be a group and $g \in G$. Define $\phi_g : G \rightarrow G$ and $\psi_g : G \rightarrow G$ such that $\phi_g(x) = xg$ and $\psi_g(x) = gx$, $x \in G$ prove that ϕ_g is one one and onto and $\phi_{gh} = \phi_h\phi_g \forall g, h \in G$. **6**
Also show that $\phi_g\psi_h = \psi_h\phi_g \forall g, h \in G$.

d) Let G be a group and ϕ is an automorphism of G . If $a \in G$ is of finite order, $O(a) > 0$, **4+**
then prove that $O(\phi(a)) = O(a)$ further show that $O(bab^{-1}) = O(a) \forall b \in G$. **2**

UNIT - II

2. a) Let R is a ring with zero element 0 . Let $a, b, c \in R$ then prove that **2+**
i) $a0 = 0a = 0$ **2+**
ii) $a(-b) = (-a)b = -(ab)$ **2**
iii) $a(b-c) = ab - ac$

b) Show that the commutative ring D is an integral domain iff for $a, b, c \in D$ with $a \neq 0$, the relation $ab = ac$ implies that $b = c$. **6**

OR

c) Define the ideal of a Ring. If U and V are ideals of a ring R then prove that **2+**
 $U + V = \{x + y \mid x \in U, y \in V\}$ is smallest ideal containing both the ideals U and V . **4**

d) If $(R, +, \cdot)$ is a ring such that $x \cdot x = x, \forall x \in R$ then show that **2+**
i) $x + x = 0, \forall x \in R$ **2+**
ii) $x + y = 0, \Rightarrow x = y; x, y \in R$ **2**
iii) R is commutative ring.

UNIT - III

3. a) Prove that $P_n(x) = \frac{1}{2^n \cdot n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ 6

b) Prove that $n P_n(x) = x P_n'(x) - P_{n-1}'(x)$ where $P_n' = \frac{dP_n}{dx}$ 6

OR

c) Prove that $x J_n'(x) = n J_n(x) - x J_{n+1}(x)$ 6

d) Prove that $4 J_n''(x) = J_{n-2}(x) - 2 J_n(x) + J_{n+2}(x)$ 4+
Hence deduce that $2 J_0''(x) = J_2(x) - J_0(x)$ 2

UNIT - IV

4. a) Find the Fourier expansion for e^x in the interval $-\pi < x < \pi$. 6

b) Find Fourier series for $f(x) = \pi x$, $-\pi < x < \pi$. 4+
2

OR

c) Express $f(x) = x$ as a cosine series in $(0, 2)$ 6

d) Obtain half range Fourier sine series for $f(x) = \ell x - x^2$, $(0, \ell)$. 6

5. Attempt **any six**.

a) Let $f : Z \rightarrow Z$ such that $f(x) = 2x$, $\forall x \in Z$, show that f is an isomorphism but not an automorphism of the addition group Z . 2

b) Show that for an abelian group G , $I(G) = \{I\}$. 2

c) Define division ring. 2

d) If, R has a unit element 1 then prove that $(-1)a = -a$. 2

e) Evaluate $\int_{-1}^1 [P_3(x)]^2 dx$ 2

f) Show that $J_1 + J_3 = \frac{4}{x} J_2$ 2

g) $f(x)$ is a periodic function with period 2ℓ defined in $c < x < c + 2\ell$ then write the Fourier coefficients a_n and b_n . 2

h) $f(x) = \sin x$, $-\pi < x < \pi$ then find b_n . 2
