B.Sc. I (With Credits)-Regular-Semester 2012 Sem II

2SMAT 104 - Mathematics -II (Differential Equations and Analysis)

Paper - IV

P. Pages : 2 **GUG/W/16/5584**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Solve $\frac{dy}{dx} + \frac{u}{x} = x^2$, given y = 1 when x = 1. **6**

b) Solve $\frac{dy}{dx} + \frac{u}{x} = y^2$ **6**

OR

c) Solve $(p - xy)(p - x^2)(p - y^2) = 0$. **6**

d) Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. **6**

UNIT - II

2. a) Solve $(x^2D^2 - 3xD + 5)y = x^2 \sin(\log x)$ **6**

b) Solve the differential equation $(D^3 - 7D - 6)\, y = e^{2x}(1 + x)$. **6**

OR

c) Explain the method variation by parameters. **6**

d) Find a particular solution of $y'' + y = \operatorname{cosec} x$ by the method of variation of parameters. **6**

UNIT - III

3. a) Define convergent sequence. Then prove that a convergent sequence of real numbers is bounded. **6**

b) Show that $\lim_{n \to \infty} \left[\left(\frac{2}{1}\right)^1 \left(\frac{3}{2}\right)^2 \left(\frac{4}{3}\right)^3 \ldots\ldots \left(\frac{n+1}{n}\right)^n \right]^{1/n} = e$ **6**

OR

c) Show that the sequence $\langle S_n \rangle$ defined by $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + ... + \frac{1}{n+n}$ converges. **6**

d) Define Cauchy sequence. Then prove that every convergent sequence of real numbers is a Cauchy sequence. **6**

UNIT - IV

4. a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^3 - n + 1}{n!}$ by ratio test. **6**

b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$ by Cauchy's integral test. **6**

OR

c) Test the convergence of the series $\sum \left(\frac{n}{n+1} \right)^n x^n, \; x > 0$ by Cauchy's root test. **6**

d) Define absolutely convergent. Show that an absolutely convergent series is convergent. **6**

5. Solve **any six.**

a) Find integrating factor of the linear differential equation $\left(1 + x^2\right) \frac{dy}{dx} + 2xy = \cos x$ **2**

b) State the condition for differential equation $M(x,\; y)dx + N(x,\; y)dy = 0$ to be exact. **2**

c) Solve $(D-1)(D+2)y = 0$ **2**

d) Solve $\frac{1}{D^2 + 1} \cos 2x$ **2**

e) Define Cauchy sequence. **2**

f) Evaluate lim S_n of the sequence $S_n = \frac{n^2 + 5n - 3}{n+5}$ **2**

g) Test the convergence of the series $u_n = \frac{n+1}{n+2}$ **2**

h) Define Alternating series. **2**

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