B.Sc. I (With Credits)-Regular-Semester 2012 Sem. I

# MAT 101 - Mathematics-I (Algebra and Trigonometry)

## Paper – I

P. Pages : 2 **GUG/W/16/3317**

Time : Three Hours * 4 6 1 7 * Max. Marks : 60

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Notes :
1. Solve **all five** questions.
2. Question 1 to 4 have alternatives solve each question in full or its alternative in full.
3. All question carry equal marks.

**UNIT – I**

**1.** a) State and prove De-Moiver's theorem for positive and negative integer. **6**

b) If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$ then prove that $x_1.x_2.x_3.......... = \cos\pi = -1$. **6**

**OR**

c) If $\sin(\alpha + i\beta) = x + iy$ then prove that **6**

i) $\frac{x^2}{\cosh^2\beta} + \frac{y^2}{\sinh^2\beta} = 1$

ii) $\frac{x^2}{\sin^2\alpha} - \frac{y^2}{\cos^2\alpha} = 1$

d) Separate into real and imaginary ports $\tan(x + iy)$. **6**

**UNIT – II**

**2.** a) Find the rank of the matrix. **6**

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

b) Find the solution of the system $x + y - z = 1,\ 3x - 2y + z = 7,\ 2x + 3y - z = 3$ by matrix method. **6**

**OR**

c) Show that if B be an invertible matrix of the same order as A, then the matrices A and $B^{-1}AB$ have the same characteristic roots. **6**

d) Verify Cayley – Hamilton theorem for the matrix. **6**

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}.$$

## UNIT – III

**3.** a) Prove that in an equation with real coefficients complex roots occures in pairs. **6**

b) Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$ roots being in arithmetical progression. **6**

**OR**

c) If a, b, c are the roots of equation $x^3 + px^2 + qx + r = 0$, then find the equation whose roots are ab, bc and ca. **6**

d) Solve the cubic equation **6**
$x^3 + 6x^2 + 9x + 4 = 0$
by Cardon's method.

## UNIT – IV

**4.** a) Prove by induction method that $3^{2n} - 1$ is divisible by 8 for all $n \in N$. **6**

b) If (a, b) is g.c.d. and [a, b] is L.C.M. of integers then prove that $(a, b).[a, b] = ab$. **6**

**OR**

c) Let a and b be integers that are not both zero. Then prove that a and b are relatively prime iff there exist integers m and n such that $ma + nb = 1$. **6**

d) Find the positive integers a and b satisfying the equations (a, b) = 10 and [a, b] = 100 simultaneously. Find all the solutions. **6**

**5.** Attempt **any six.**

a) Prove that, **2**
$(\sin\theta + i\cos\theta)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$.

b) Express $(1+i)$ in Polar form. **2**

c) State Cayley – Hamilton theorem. **2**

d) Find the characteristic equation of matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. **2**

e) Find the nature of the roots of equation $3x^4 + 12x^2 + 5x - 4 = 0$. **2**

f) Find the equation whose roots are the roots of $x^7 + 3x^5 + x^3 - x^2 + 7x + 2 = 0$ with their sign changed. **2**

g) Let a, b, c be integers such that a/b and b/c then prove that a/c. **2**

h) Prove that **2**
$(a, bc) = 1 \Rightarrow (a, b) = 1$ and $(a, c) = 1$.

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